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BULK ARRIVAL RETRIAL QUEUE WITH FLUCTUATING MODES OF SERVICE, IMMEDIATE FEEDBACK AND SERVER VACATION

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ABSTRACT

Batch arrival retrial queue with immediate feedback is considered. Server provides M modes of service to the customers. If an arriving batch of customers finds the server free, one of the arrivals chooses any one of the M modes of service immediately and the rest join the orbit. Otherwise all the arriving customers join the orbit. At a service completion epoch, the unsatisfied customers opt for re-service. After completion of each service, the server departs for a single vacation of arbitrary distributed length according to Bernoulli schedule. Analytical treatment of this model is obtained by the supplementary variable technique. Stochastic decomposition law is verified. Numerical results are presented.

***Keywords*:** Retrial Queue, Fluctuating Modes, Immediate Feedback and Vacation.

I. INTRODUCTION

Queueing systems with repeated attempts are found suitable for modeling the processes in telephone switching systems, digital cellular model networks, packet switching networks, local area networks, stock and flow etc. Review of retrial queueing literature can be found in the survey papers by Yang and Templeton (1987), Falin (1990), Kulkarni and Liang (1997) and Artalejo (1999a, 1999b, 2010), the book by Falin and Templeton (1997) and the monographs by Artalejo and Gomez-Corral (2008).

Single server queueing system in literature assumes that the server provides one type of general service with same mean rate to all the customers. But in real life situations there could be variations in mean service rate due to variety of reasons. Baruah et al. (2014) discussed a queueing system in which the server provides general service in two fluctuating modes. Madan (2014) studied a queueing system in which the server provides general service to customers in three fluctuating modes with different service rates and obtained probabilities of the idle states as well as the utilization factor of the system explicity.

Re-Service have many real life applications in ATM machines, bank counters, super markets, doctor clinics, etc. Madan and Baklizi (2002) analysed an M/G/1 queue with additional second stage service and optional re-service. Madan et al. (2004) discussed on $M^{[x]}/(G_1, G_2)/1$ queue with optional re-service. Baruah et al. (2012) investigate balking and re-service in a vacation queue with batch arrival and two types of heterogeneous service. Rajadurai et al.

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(2014) studied an unreliable $M^{[X]}/(G_1,G_2)/1$ retrial queueing system with balking, optional re-service under modified vacation policy using supplementary variable technique. The customers opting for re-service immediately after a service completion has been dealt only by few authors. The present paper investigates bulk arrival retrial queue with fluctuating modes of service, immediate feedback (re-service) and Bernoulli vacation.

II. MODEL DESCRIPTION

Consider a single server retrial queueing system in which customers arrive in batches according to a compound Poisson process with rate λ. The batch size Y is a random variable with distribution function $P(Y=k) = C_k$, k=1,2,..., and probability generating function C(z) having first two moments m_1 and m_2. The server provides M heterogeneous modes of service and the probability of providing mode i service is $p_i (1 \le i \le M)$. If an arriving batch finds the server free, one of the customers in the batch begins any one of the M modes and the rest join the orbit. Inter-retrial times have an arbitrary distribution function A(x), density function a(x), Laplace –Stieltje's transform $A^*(s)$ and conditional completion rate $\eta(x) = \frac{a(x)}{[1-A(x)]}$.

The service time of mode i (i= 1,2,…,M) follows a general distribution with distribution function $B_i(x)$, density function $b_i(x)$, Laplace–Stieltje's transform $B_i^*(s)$, n^{th} factorial moments $\mu_{i,n}$ and conditional completion rate $\mu_i(x) = \frac{b_i(x)}{[1-B_i(x)]}$.

At the completion of each service the server takes a single vacation with probability τ or waits for the next customer with complementary probability $1-\tau$. The vacation time is generally distributed with distribution function V(x), density function v(x), Laplace Stieltje's transform $V^*(s)$, n^{th} factorial moments v_n and conditional completion rate $\gamma(x) = \frac{v(x)}{[1-V(x)]}$.

After completion of mode i service, the customer may opt for the same service with probability r_i or leave the system with its complementary probability $(1- r_i)$. In this case it is assumed that the customers are allowed to repeat the service only once.

III. STEADY STATE DISTRIBUTION

Let N (t) denote the number of customers in the orbit at time t and C(t) denote the state of the server defined as

$$C(t) = \begin{cases} 0, & \text{if the server is idle} \\ i, & \text{if the server is busy in mode i service} \\ M+i, & \text{if the server is busy in mode i re-service} \\ 2M+1, & \text{if the server is on vacation} \end{cases}$$

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The state of the system at time t can be described by the Markov process $\{X(t); t \geq 0\} = \{C(t), N(t), \xi_0(t), \xi_1(t), \xi_2(t), \xi_3(t); t \geq 0\}$. If C(t)=0, then $\xi_0(t)$ represents the elapsed retrial time, if C(t)=i$(1 \leq i \leq M)$ $\xi_1(t)$ represents the elapsed service time, if C(t)=M+i $\xi_2(t)$ represents the elapsed re-service time and if C(t)=2M+1 $\xi_3(t)$ represents the elapsed vacation time.

Define the following probability densities

$I_0(t) \quad = P\{C(t) = 0, N(t) = 0\}$

$I_n(x,t)dx \ = P\{C(t) = 0, N(t) = n, x \leq \xi(t) < x + dx\}$, $x \geq 0$, $n \geq 1$

$P_{i,n}(x,t)dx = P\left\{C(t) = i, N(t) = n, x \leq \xi(t) < x + dx\right\}, x \geq 0, n \geq 0,\ i = 1,2,\ldots,M.$

$Q_{i,n}(x,t)dx = P\left\{C(t) = M + i, N(t) = n, x \leq \xi(t) < x + dx\right\}, x \geq 0, n \geq 0, i = 1,2,\ldots,M.$

$V_n(x,t)dx = P\{C(t) = 2M + 1, N(t) = n, x \leq \xi(t) < x + dx\}$, $x \geq 0$, $n \geq 0$

Let I_0, $I_n(x)$, $P_{i,n}(x)$, $Q_{i,n}(x)$ and $V_n(x)$ be the steady state probabilities of $I_0(t)$, $I_n(x,t)$, $P_{i,n}(x,t)$, $Q_{i,n}(x,t)$ and $V_n(x,t)$, where $n \geq 0$, $x \geq 0$, $i = 1,2,\ldots,M$.
The system of equilibrium equations governing the model is given below

$$\lambda I_0 = (1-\tau)\left[\sum_{i=1}^{M}\left((1-r_i)\int_0^\infty P_{i,0}(x)\mu_i(x)dx\right) + \int_0^\infty Q_{i,0}(x)\mu_i(x)dx\right] + \int_0^\infty V_0(x)\gamma(x)dx \tag{1}$$

$$\frac{d}{dx} I_n(x) = -(\lambda + \eta(x)) I_n(x), \quad n \geq 1 \tag{2}$$

$$\frac{d}{dx} P_{i,n}(x) = -(\lambda + \mu_i(x)) P_{i,n}(x) + \lambda \sum_{k=1}^{n} c_k P_{i,n-k}(x), \quad n \geq 0, \ i = 1,2,\ldots,M \tag{3}$$

$$\frac{d}{dx} Q_{i,n}(x) = -(\lambda + \mu_i(x)) Q_{i,n}(x) + \lambda \sum_{k=1}^{n} c_k Q_{i,n-k}(x), \quad n \geq 0, \ i = 1,2,\ldots,M \tag{4}$$

$$\frac{d}{dx} V_n(x) = -(\lambda + \gamma(x)) V_n(x) + \lambda \sum_{k=1}^{n} C_k V_{n-k}(x), \ n \geq 0 \tag{5}$$

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with boundary conditions

$$I_n(0) = (1-\tau)\left[\sum_{i=1}^{M}\left((1-r_i)\int_0^\infty P_{i,n}(x)\mu_i(x)\,dx\right)+\int_0^\infty Q_{i,n}(x)\mu_i(x)\,dx\right] + \int_0^\infty V_n(x)\gamma(x)dx,\ n\ge 1 \tag{6}$$

$$P_{i,0}(0) = p_i\left[\lambda c_1 I_0 + \int_0^\infty I_1(x)\eta(x)\,dx\right],\ i=1,2,\ldots,M \tag{7}$$

$$P_{i,n}(0) = p_i\left[\lambda c_{n+1} I_0 + \int_0^\infty I_{n+1}(x)\eta(x)\,dx + \lambda\sum_{k=1}^{n} c_k\int_0^\infty I_{n-k+1}(x)\,dx\right],\quad n\ge 1,\ i=1,2,\ldots,M \tag{8}$$

$$Q_{i,n}(0) = r_i\int_0^\infty P_{i,n}(x)\mu_i(x)\,dx,\ n\ge 1,\ i=1,2,\ldots,M \tag{9}$$

$$V_n(0) = \tau\sum_{i=1}^{M}\left\{\left((1-r_i)\int_0^\infty P_{i,n}(x)\mu_i(x)\,dx\right)+\int_0^\infty Q_{i,n}(x)\mu_i(x)\,dx\right\},\ n\ge 1 \tag{10}$$

The normalizing condition is

$$I_0+\sum_{n=1}^{\infty}\int_0^\infty I_n(x)\,dx+\sum_{n=0}^{\infty}\left[\sum_{i=1}^{M}\int_0^\infty P_{i,n}(x)\,dx+\int_0^\infty Q_{i,n}(x)\,dx\right]+\sum_{n=1}^{\infty}\int_0^\infty V_n(x)\,dx=1 \tag{11}$$

Define the probability generating functions

$$I(x,z)=\sum_{n=1}^{\infty} I_n(x)z^n;\ P_i(x,z)=\sum_{n=0}^{\infty}P_{i,n}(x)z^n;\ Q_i(x,z)=\sum_{n=0}^{\infty}Q_{i,n}(x)z^n;\ i=1,2,\ldots,M$$

$$\text{and } V(x,z)=\sum_{n=0}^{\infty}V_n(x)z^n$$

Multiplying equations (2) to (10) by z^n and summing over for all possible values of n, we obtain the following results

$$\left[\frac{d}{dx}+(\lambda+\eta(x))\right]I(x,z)=0 \tag{12}$$

$$\left[\frac{d}{dx}+\lambda(1-c(z))+\mu_i(x)\right]P_i(x,z)=0,\quad i=1,2,\ldots,M \tag{13}$$

$$\left[\frac{d}{dx}+\lambda(1-c(z))+\mu_i(x)\right]Q_i(x,z)=0,\quad i=1,2,\ldots,M \tag{14}$$

$$\left[\frac{d}{dx}+\lambda(1-c(z))+\gamma(x)\right]V(x,z)=0 \tag{15}$$

$$\begin{aligned}I(0,z) &= (1-\tau)\left\{\sum_{i=1}^{M}\left((1-r_i)\int_0^\infty P_i(x,z)\mu_i(x)\,dx\right)+\int_0^\infty Q_i(x,z)\mu_i(x)\,dx\right\}\\ &\quad+\int_0^\infty V(x,z)\gamma(x)\,dx-\lambda I_0\end{aligned} \tag{16}$$

$$P_i(0,z) = \frac{p_i}{z}\left[\lambda c(z)I_0+I(0,z)\left(A^*(\lambda)+c(z)\left(1-A^*(\lambda)\right)\right)\right] \tag{17}$$

$$Q_i(0,z)= r_i P_i(0,z)B_i^*(h(z)) \tag{18}$$

$$V(0,z) = \tau\left\{\sum_{i=1}^{M}P_i(0,z)B_i^*(h(z))\left[1-r_i+r_i B_i^*(h(z))\right]\right\} \tag{19}$$

where

$$h(z) = \lambda-\lambda c(z)$$

Solving the partial differential equations (12) to (15), we get

$$I(x,z) = I(0,z)e^{-\lambda x}(1-A(x)) \tag{20}$$

$$P_i(x,z) = P_i(0,z)e^{-[\lambda(1-c(z))]x}(1-B_i(x)) \tag{21}$$

$$Q_i(x,z) = Q_i(0,z)e^{-[\lambda(1-c(z))]x}(1-B_i(x)) \tag{22}$$

$$V(x,z) = V(0,z)e^{-[\lambda(1-c(z))]x}(1-V(x)) \tag{23}$$

Using equations (17), (18), (19), (21), (22) and (23) in equation (16) and simplifying, we get

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$$I(0,z)=\lambda I_0\left[C(z)T_1(z)\left(1-\tau+\tau V^*(h(z))\right)-z\right]\Big/D(z) \tag{24}$$

where

$$T_1(z)=\sum_{i=1}^{M} p_i B_i^*(h(z))\left(1-r_i+r_i B_i^*(h(z))\right)$$

$$D(z)=z-\left[A^*(\lambda)+C(z)\left(1-A^*(\lambda)\right)\right]T_1(z)\left(1-\tau+\tau V^*(h(z))\right)$$

Using equation (24), the equation (17) becomes

$$P_i(0,z)=\lambda I_0 A^*(\lambda)p_i[c(z)-1]\Big/D(z),\ i=1,2,\ldots,M \tag{25}$$

Inserting equation (25) in equation (18) and (19), we get

$$Q_i(0,z)=\lambda I_0 A^*(\lambda)p_i r_i B_i^*(h(z))[c(z)-1]\Big/D(z),\ i=1,2,\ldots,M \tag{26}$$

$$V(0,z)=\lambda I_0 A^*(\lambda)(C(z)-1)\tau T_1(z)\Big/D(z) \tag{27}$$

Substituting the expressions of $I(0,z)$, $P_i(0,z)$, $Q_i(0,z)$ and $V(0,z)$ in equations (20), (21), (22) and (23), we get the following results

$$I(x,z)=\lambda I_0\left[C(z)T_1(z)\left(1-\tau+\tau V^*(h(z))\right)-z\right]e^{-\lambda x[1-A(x)]}\Big/D(z) \tag{28}$$

$$P_i(x,z)=\lambda I_0 A^*(\lambda)p_i[C(z)-1]e^{-(h(z))x}[1-B_i(x)]\Big/D(z),\ i=1,2,\ldots,M \tag{29}$$

$$Q_i(x,z)=\lambda I_0 A^*(\lambda)r_i p_i B_i^*(h(z))[C(z)-1]e^{-(h(z))x}[1-B_i(x)]\Big/D(z),\quad i=1,2,\ldots,M \tag{30}$$

$$V(x,z)=\lambda I_0 A^*(\lambda)(C(z)-1)\tau T_1(z)e^{-(h(z))x}[1-V(x)]\Big/D(z) \tag{31}$$

The partial probability generating function of the orbit size when the server is idle is given by

$$I(z) = \int_0^{\infty} I(x,z)dx$$

$$= I_0 \left(1 - A^*(\lambda)\right)\left[C(z)\ T_1(z)\ \left(1-\tau+\tau V^*(h(z))\right) - z\right] \Big/ D(z) \tag{32}$$

The partial probability generating function of the orbit size when the server is busy is given by

$$B(z) = \sum_{i=1}^{M}[P_i(z) + Q_i(z)]$$

$$= I_0\ A^*(\lambda) \sum_{i=1}^{M} p_i \left(B_i^*(h(z)) - 1\right)\left[1 + r_i B_i^*(h(z))\right] \Big/ D(z),\ i = 1, 2, \ldots, M \tag{33}$$

The partial probability generating function of the orbit size when the server is on vacation is given by

$$V(z) = \int_0^{\infty} V(x,z)\,dx$$

$$= I_0\ A^*(\lambda)\ \tau\ T_1(z)\left[V^*(h(z)) - 1\right] \Big/ D(z) \tag{34}$$

Probability generating function of the number of customers in the orbit is given by

$$P_q(z) = I_0 + I(z) + \sum_{i=1}^{M} P_i(z) + \sum_{i=1}^{M} Q_i(z)$$

$$= I_0\ A^*(\lambda)[z-1] \Big/ D(z) \tag{35}$$

Probability generating function of the number of customers in the system is given by

$$P_s(z) = I_0 + I(z) + z\sum_{i=1}^{M} P_i(z) + z\sum_{i=1}^{M} Q_i(z)$$

$$= I_0\ A^*(\lambda)\,[z-1]\ T_1(z) \Big/ D(z) \tag{36}$$

IV. PERFORMANCE MEASURES

- Probability that the server is idle is given by

$$I = \lim_{z \to 1} I(z)$$

$$= I_0\left(1 - A^*(\lambda)\right)\left[m_1 - 1 + \lambda m_1 \sum_{i=1}^{M} p_i\, \mu_{i1}\left(r_i + 1\right) + \tau\ \lambda m_1 v_1 \sum_{i=1}^{M} p_i\right] \Big/ D'(1)$$

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(37)

where

$$D^{'}(1) = 1 - m_1\left(1 - A^{*}(\lambda)\right) - \lambda m_1 \sum_{i=1}^{M} p_i \mu_{i1}\left(r_i + 1\right)$$

- Probability that the server is busy is given by

$$B = \lim_{Z \to 1} B(z)$$

$$= I_0 A^{*}(\lambda)\, \lambda m_1 \sum_{i=1}^{M} p_i \mu_{i1}\left(1 + r_i\right) \Big/ D^{'}(1) \qquad (38)$$

- Probability that the server is on vacation is given by

$$V = \lim_{z \to 1} V(z)$$

$$= I_0\, A^{*}(\lambda)\, \tau\, \lambda m_1 v_1 \sum_{i=1}^{M} p_i \Big/ D^{'}(1) \qquad (39)$$

Using normalizing condition I_0 is obtained as

$$I_0 = \frac{1 - m_1\left(1 - A^{*}(\lambda)\right) - \lambda m_1 \sum_{i=1}^{M} p_i \mu_{i1}\left(r_i + 1\right) - \tau \lambda m_1 v_1 \sum_{i=1}^{M} p_i}{A^{*}(\lambda)} \qquad (40)$$

Mean number of customers in the orbit L_q under steady state condition is given by

$$L_q = \lim_{z \to 1} \frac{d}{dz} P_q(z)$$

$$= \frac{D^{'}(1) N^{''}(1) - N^{'}(1) D^{''}(1)}{2 D^{'}(1)^{2}} \qquad (41)$$

where N(z) and D(z) are the Numerator and Denominator of $P_q(z)$.

$$N^{'}(1) = I_0\, A^{*}(\lambda)$$

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$$D''(1) = -m_2\left(1-A^*(\lambda)\right) - 2\tau\lambda m_1^2 v_1\left(1-A^*(\lambda)\right)\sum_{i=1}^{M} p_i - 2\lambda^2 m_1^2 \sum_{i=1}^{M} p_i \mu_{i1}^2 r_i$$
$$-\lambda^2 m_1^2 \sum_{i=1}^{M} p_i \mu_{i2}\left(r_i+1\right) - \lambda m_2 \sum_{i=1}^{M} p_i \mu_{i1}\left(r_i+1\right) - \tau\lambda^2 m_1^2 v_2 \sum_{i=1}^{M} p_i$$
$$-\tau\lambda m_2 v_1 \sum_{i=1}^{M} p_i - 2\lambda m_1^2\left(1-A^*(\lambda)\right)\sum_{i=1}^{M} p_i \mu_{i1}\left(r_i+1\right)$$
$$-2\lambda^2 m_1^2 \tau v_1 \sum_{i=1}^{M} p_i \mu_{i1}\left(r_i+1\right)$$

The mean number of customers in the system is given by

$$L_s = \lim_{z\to 1}\frac{d}{dz}P_s(z)$$

$$= L_q + B \tag{42}$$

V. STOCHASTIC DECOMPOSITION

Theorem:

The number of customers in the system (L_s) under steady state can be expressed as the sum of two independent random variables one of which is the mean number ofcustomers (L) in the classical batch arrival queueing system with fluctuating modes of service, immediate feedback and other is the mean number of customers in the orbit (L_I) given that the server is idle or on vacation.

Proof:

The probability generating function, $\pi(z)$ of the system size in the classical batch arrival queue with fluctuating modes of service and immediate feedback is given by

$$\pi(z) = \frac{\left(1-\lambda m_1 \sum_{i=1}^{M} p_i \mu_{i1}\left(r_i+1\right)\right)\left[(z-1)\; T_1(z)\right]}{z - T_1(z)} \tag{43}$$

The probability generating function, $\psi(z)$ of the number of customers in the orbit when the system is idle or on vacation is given by

$$\psi(z) = \frac{I_0 + I(z) + V(z)}{I_0 + I + V}$$

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$$= \frac{[z - T_1(z)]\, D'(1)}{\left[1 - \lambda m_1 \sum_{i=1}^{M} p_i \mu_{i1} (r_i + 1)\right] D(z)} \tag{44}$$

From equations (36),(43) and (44) ,we see that

$$P_s(z) = \pi(z).\ \psi(z) \tag{45}$$

Differentiating (45) with respect to z and taking limit as z→1, we get

$$L_s = L + L_I$$

VI. NUMERICAL RESULTS

Performance measures are calculated numerically by assuming that the retrial time, service time and vacation time follow exponential distribution with respective rates η, μ_1, μ_2 and γ.

For the parameters $\lambda = 0.3$, $\eta = 5$, $\mu_1 = 12$, $\mu_2 = 13$, $r_1 = 0.4$, $r_2 = 0.4$, $p_1 = 0.7$, $p_2 = 0.3$, $m_1 = 1.5$, $m_2 = 1$, $\gamma = 5$, the performance measures I_0 – the probability that the system is empty, I – the probability that the server is idle in non-empty system, V – the probability that the server is on vacation and L_s – the mean number of customers in the system are calculated and displayed in Fig. 1.1 and Fig.1.2.

From the Figures, it is observed that

- $I_0 + I$ decreases with increase in λ and μ_1 and independent of η and τ.
- V increases with increase in λ and μ_1 and is independent of η and τ.
- L_s increases with increase in λ and μ_1 and decreases with increase in η and τ.

The values of $I_0 + I$, V and L_s by varying the parameters η, μ_1 and τ are calculated and presented in Table.

- $I_0 + I$ increases with increase in μ_1, decreases with increase in τ and independent of η.
- V increases with increase in τ and is independent of η and μ_1.
- L_s increases with increase in τ and decreases with increase in η and μ_1.

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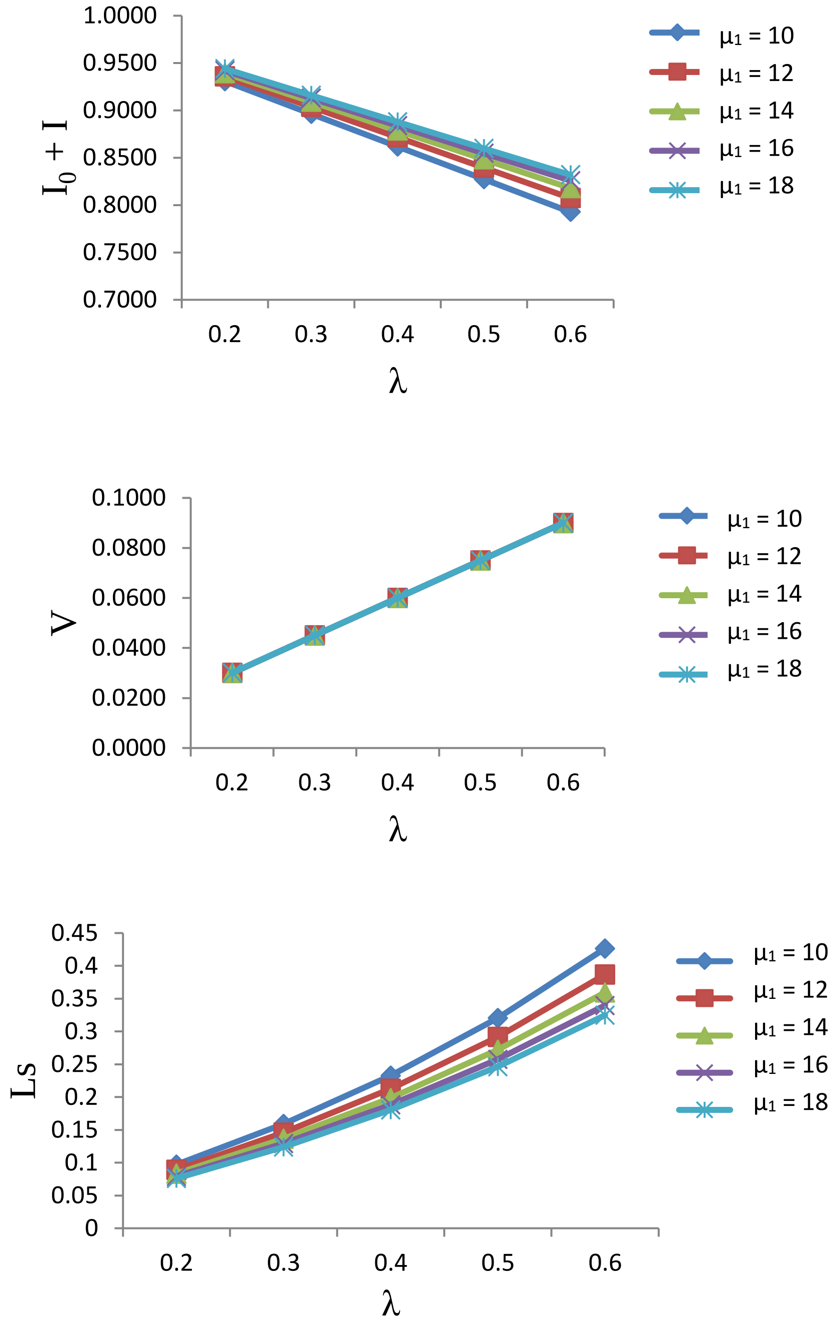

Fig. 1.1 Effect of (λ, μ_1) on $I_0 + I$, V and L_S

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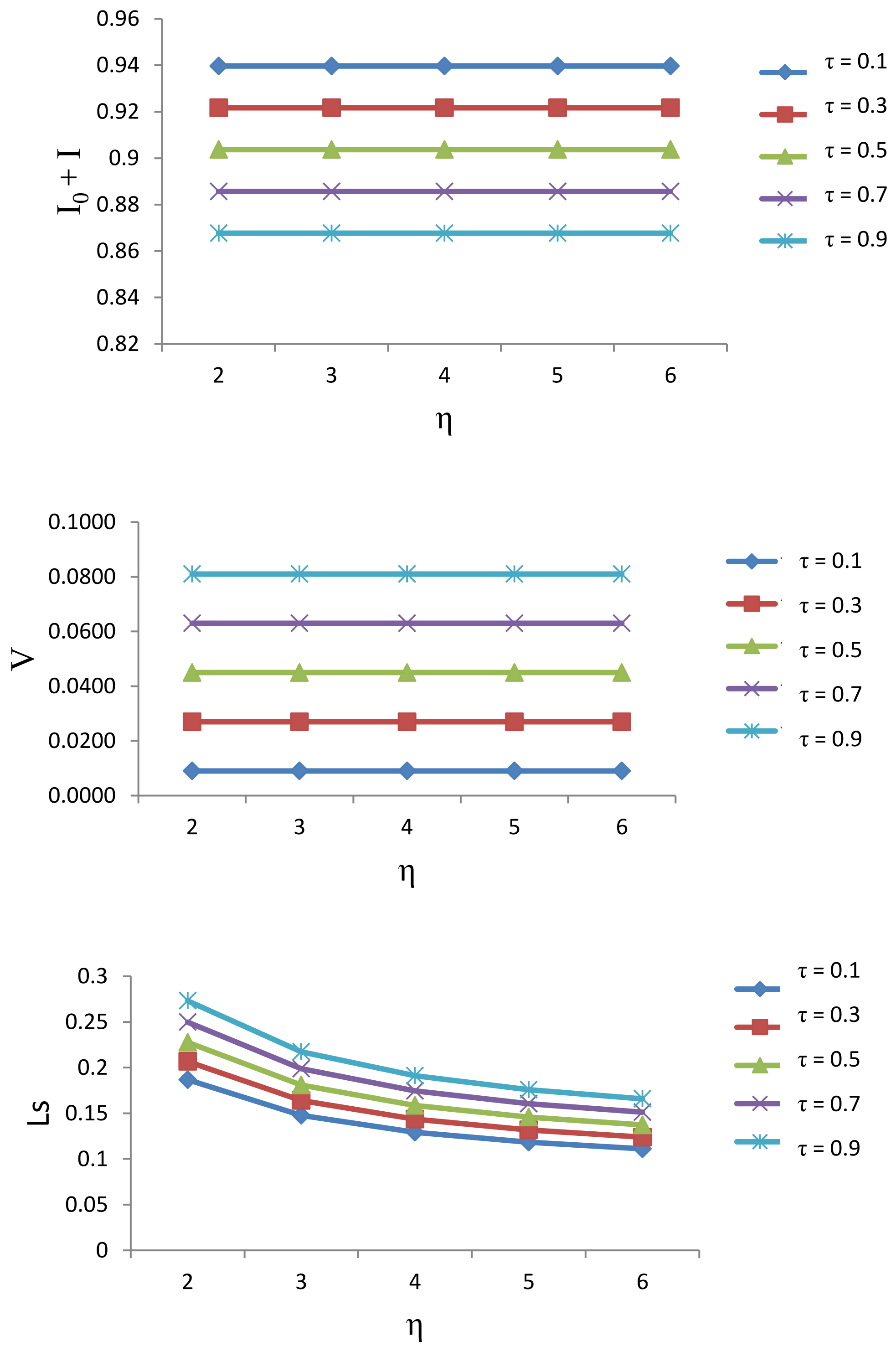

Fig. 1.2 Effect of (η, τ) ***on*** $I_0 + I$***, V and*** L_S

Table: Performance Measures Versus by η, μ_1 and τ

η	μ_1	τ	$I_0 + I$	V	L_s
2	12	0.1	0.9397	0.0090	0.1868
		0.5	0.9037	0.0450	0.2277
		0.9	0.8677	0.0810	0.2731
	14	0.1	0.9450	0.0090	0.1762
		0.5	0.9090	0.0450	0.2163
		0.9	0.8730	0.0810	0.2607
3	12	0.1	0.9397	0.0090	0.1477
		0.5	0.9037	0.0450	0.1809
		0.9	0.8677	0.0810	0.2174
	14	0.1	0.9450	0.0090	0.1381
		0.5	0.9090	0.0450	0.1707
		0.9	0.8730	0.0810	0.2064
4	12	0.1	0.9397	0.0090	0.1291
		0.5	0.9037	0.0450	0.1587
		0.9	0.8677	0.0810	0.1911
	14	0.1	0.9450	0.0090	0.1200
		0.5	0.9090	0.0450	0.1490
		0.9	0.8730	0.0810	0.1807

VII. CONCLUSION

In this paper bulk arrival retrial queue with fluctuating modes of service, immediate feedback and Bernoulli vacation is studied. Using supplementary variable technique probability generating functions of the server state and other performance measures in steady state are obtained. Numerical examples have been carried out to illustrate the effect of varying parameters on the performance measures.

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